# NOTE. GEOMETRICAL PROOF OF THE THREE-AMMETER METHOD OF MEASURING POWER.

By Frederick Bedell and Albert C. Crehore.

THE methods, now well-known, for measuring power by three voltmeters or three ammeters, first shown by Professor Ayrton and Dr. Sumpner,[1] are applicable to the measurement of power of any circuit irrespective of the nature of the impressed electromotive force, and the general proof of the methods is given in the paper referred to. In the case of an harmonic electromotive force the methods are capable of simple geometrical proof. The writers have shown this[2] for the voltmeter method, and in this note will give the corresponding proof for the three-ammeter method.

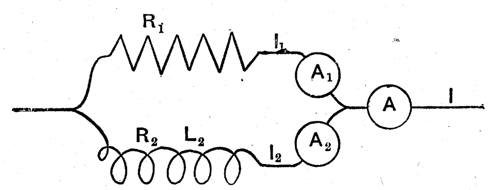

Fig. 1.

Let $R_2L_2$, Fig. 1, be an inductive circuit whose power is to be measured, and $R_1$ a non-inductive resistance in parallel with it. If the maximum values of the main and branch currents be denoted by $I$, $I_1$, and $I_2$, respectively, they may be represented as shown in Fig. 2. The current $I_1$ is in phase with the impressed electromotive force $E$; the main current, $I$, lags behind it by an angle $\theta$; and the current $I_2$ lags behind it by an angle $\theta_2$. The tangent of $\theta_2$ is $\frac{L_2\omega}{R_2}$; and the tangent of $\theta$ is $\frac{L'\omega}{R'}$,

[1] "The Measurement of Power given by Any Electric Current to Any Circuit." Proc. Roy. Soc., Vol. XLIX., p. 424.

[2] "Alternating Currents," p. 230.

where $R'$ and $L'$ denote the equivalent resistance and self-induction of the parallel circuit,[1] and $\omega$ is $2\pi$ times the frequency.

The power expended in the inductive circuit is $W_2=\frac{1}{2}EI\cos\theta_2$. From the geometry of the figure

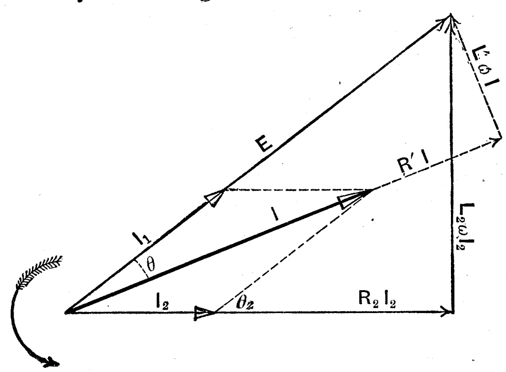


Fig. 2.

$$\cos\theta_2=\frac{I^2-I_1^2-I_2^2}{2I_1I_2};$$

whence

$$W_2=\frac{E}{4I_1}(I^2-I_1^2-I_2^2),$$

where $E$, $I$, $I_1$, and $I_2$ represent maximum values. Writing virtual or square root of mean square values as obtained from ordinary measuring instruments, the expression for the power becomes:

$$W_2=\frac{\overline{E}}{2\overline{I}_1}(\overline{I}^2-\overline{I}_1^2-\overline{I}_2^2).$$

The power in the non-inductive branch is

$$W_1=\overline{E}\overline{I}_1;$$

and the total power in the two branches is

$$W=\frac{\overline{E}}{2\overline{I}_1}(\overline{I}^2+\overline{I}_1^2-\overline{I}_2^2).$$

[1] "Equivalent Resistance, Self-Induction and Capacity of Parallel Circuits with Harmonic Impressed Electromotive Force." Bedell and Crehore, Phil. Mag., September, 1892.

The method is thus geometrically established for harmonic currents, which may be represented by lines in a vector diagram. For an alternating current, not harmonic, the proof does not hold unless we assume the current to be equivalent to an harmonic current, and the question then arises as to what will be the equivalent harmonic current. The equivalent harmonic current must be such that its square root of the mean square value, and the expenditure of energy in the circuit, are the same as in the case of the given current which was not harmonic.